The Effect of Policyholders' Rationality on Unit-Linked Life Insurance Contracts with Surrender Guarantees

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Introduction

Four existing approaches to evaluate unit-linked life insurance contracts with surrender guarantees:

- build surrender table;
- American-style contingent claim framework, e.g., Grosen and Jørgensen (1997,2000), Bacinello (2003,2005);
- suboptimal surrender, decision parameter, e.g., Bernard and Lemieux (2008);
- exogenous and endogenous surrender,e.g., Albizzati and Geman (1994), Giovanni (2010).

Introduction

Our approach is:

- similar to Albizzati and Geman (1994) and Giovanni (2010);
- but different from them in two aspects:
  - we compare the surrender value with the value of a new contract which also bears surrender risk and solve the problem dynamically;
  - we incorporate mortality risk;

Model Setup

Financial Market:

- Non-dividend paying risky asset S:

$$\mathrm{d}S_t = a(t, S_t)S_t\mathrm{d}t + \sigma(t, S_t)S_t\mathrm{d}W_t,$$

where W is the Brownian motion under the real world measure $\mathbb{P}$ and generates the financial market filtration $\mathcal{F} = (\mathcal{F}_t)_{0\leq t\leq T}$.

- Riskless money account B:

$$\mathrm{d}B_t = r(t)B_t\mathrm{d}t,$$

where the risk-free interest rate r is deterministic.

- The financial market is complete and arbitrage free, i.e., there exists a unique risk-neutral martingale measure $\mathbb{Q}$ so that

$$\mathrm{d}S_t = r(t)S_t\mathrm{d}t + \sigma(t, S_t)S_t\mathrm{d}\hat{W}_t, \quad 0 \leq t \leq T,$$

where $\hat{W}$ is a Brownian motion under $\mathbb{Q}$ which satisfies $\mathrm{d}\hat{W}_t = \mathrm{d}W_t + \frac{a-r}{\sigma}\,\mathrm{d}t$.

Model Setup

Insurance Market:

- The random time τ denotes the death time of an individual aged x at the starting time 0.
- The hazard rate of τ is denoted by μ, assumed to be deterministic.
- The jump process $H_t = 1_{\{\tau \le t\}}$ generates the filtration $\mathbb{H} = (\mathcal{H}_t)_{0 \le t \le T}$.

Combined Market under $\mathbb{Q}$:

- $\mathbb{G} = \mathbb{F} \vee \mathbb{H}$, such that $\hat{W}$ is a $(\mathbb{Q}, \mathbb{G})$-Brownian motion and μ is the $(\mathbb{Q}, \mathbb{G})$-intensity of H.

Contract Valuation

The contract is comprised of

- the survival benefit

$$\Phi(S_T) = P \max\left(\alpha(1+g)^T, \left(\frac{S_T}{S_0}\right)^k\right),$$

- the death benefit

$$\Psi(\tau, S_\tau) = P \max\left(\alpha(1+g)^\tau, \left(\frac{S_\tau}{S_0}\right)^{k_d}\right),$$

- the surrender benefit

$$L(\lambda) = (1-\beta_\lambda)P\,(1+h)^\lambda$$

Contract Valuation

The arrival of the surrender action at a random time λ is described by a generalized Poisson process with stochastic intensity γ.

$$\gamma_t = \begin{cases} \underline{\rho}, & \text{for } L(t) < V_t \\ \bar{\rho}, & \text{for } L(t) \geq V_t. \end{cases}$$

The contract value V_t on $\{t < \lambda \wedge \tau \wedge T\}$ satisfies

$$r(t) V_t \mathrm{d}t = \mathbb{E}_{\mathbb{Q}} \left[\mathrm{d}V_t | \mathcal{G}_t \right].$$

Contract Valuation

$$r(t)V_t\mathrm{d}t = \mathbb{E}_{\mathbb{Q}}\left[\mathrm{d}V_t|\mathcal{G}_t\right].$$

We consider three cases under the assumption that the two stopping times τ and λ are conditionally independent of each other:

1) The conditional probability that death occurs over $(t, t+\mathrm{d}t)$ while the surrender does not is $\mu_t\mathrm{d}t(1-\gamma_t\mathrm{d}t) = \mu_t\mathrm{d}t$.
2) The conditional probability that surrender occurs over $(t, t+\mathrm{d}t)$ while the death event has not happened is $\gamma_t\mathrm{d}t(1-\mu_t\mathrm{d}t) = \gamma_t\mathrm{d}t$.
3) The conditional probability that both the surrender and the death events occur over $(t, t+\mathrm{d}t)$ is 0.

Contract Valuation

On $\{t < \lambda \wedge \tau \wedge T\}$,

$$r(t)V_t \mathrm{d}t = \mathbb{E}_{\mathbb{Q}}\left[\mathrm{d}V_t | \mathcal{G}_t\right].$$

is equivalent to

$$\begin{aligned} r(t)v(t,S_t)\mathrm{d}t &= \mathbb{E}_{\mathbb{Q}}[\mathrm{d}v(t,S_t)|\mathcal{F}_t] + (\Psi(t,S_t) - v(t,S_t))\mu_t \mathrm{d}t \\ &+ (L(t) - v(t,S_t))\gamma(t,S_t)\mathrm{d}t. \end{aligned}$$

Then we obtain

$$\mathcal{L}v(t,s) + \mu(t)\Psi(t,s) + \gamma(t,s)L(t) - (r(t) + \mu(t) + \gamma(t,s))v(t,s) = 0$$

for $t < T$ and $S_t = s$.

Numerical Analysis

Parameters:

Underlying asset	S&P index
Volatility (σ)	0.2
Interest rate (r)	0.04
Initial investment (P)	100
Maturity (T)	10 years
Participation rate (α)	0.85
Minimum guarantee at survival (g)	0.02
Minimum guarantee at death (g_d)	0.02
Minimum guarantee at surrender (h)	0.02
Participation coefficient at survival (k)	0.9
Participation coefficient at death (k_d)	0.9
Penalty rates (β)	$\beta_1 = 0.05$, $\beta_2 = 0.04$, $\beta_3 = 0.02$ $\beta_4 = 0.01$, $\beta_t = 0$ for $t \geq 5$
mortality intensity ($\mu(t)$)	$\mu(t) = A + B\,c^{x+t}$ with $A = 5.0758 \times 10^{-4}$, $B = 3.9342 \times 10^{-5}$, $c = 1.1029$ and $x = 40$

Numerical Analysis

	$\bar{\rho}$				
$\underline{\rho}$	0.00	0.03	0.30	3.00	∞
0	102.7630	103.9335	108.2971	110.6107	110.9602
0.03	-	99.4447	103.5910	105.5440	105.8250
0.3	-	-	92.7071	94.4926	94.9999

Table 1: Contract value V_0 for various bounds $\underline{\rho}$ and $\bar{\rho}$ for the surrender intensity.

Numerical Analysis

The Separating Boundary

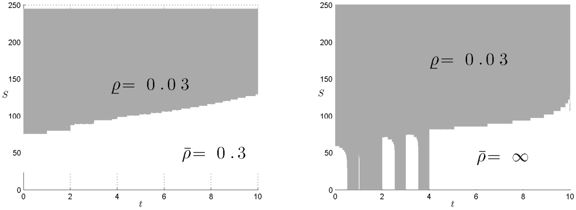

Figure 1: The separating boundary ∂C for $\underline{\rho} = 0.03$, $\bar{\rho} \in \{0.3, \infty\}$, $\alpha = 0.85$, $g = g_d = h = 0.02$, $k = k_d = 0.9$, $\beta_1 = 0.05$, $\beta_2 = 0.04$, $\beta_3 = 0.02$, $\beta_4 = 0.01$ and $\beta_t = 0$ for $t \geq 5$.

Numerical Analysis

The Separating Boundary

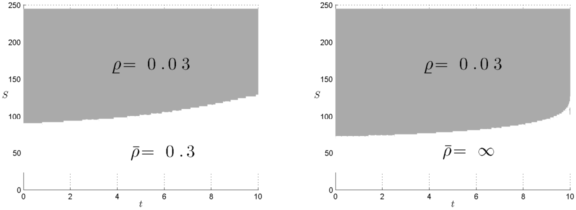

Figure 2: The separating boundary of ∂C for $\rho = 0.03$, $\bar{\rho} \in \{0.3, \infty\}$, $\alpha = 0.85$, $g = g_d = h = 0.02$, $k = k_d = 0.9$, $\beta_t = 0$ for $t \geq 0$.

Numerical Analysis

Fair Contract Analysis

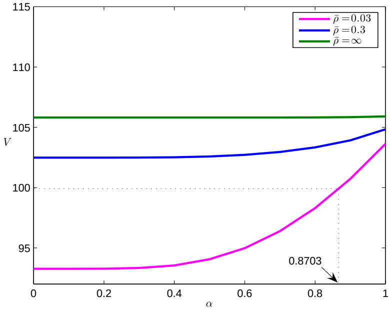

Figure 3: The contract value V_0 depending on the participation rate in the minimum guarantee α for $\underline{\rho} = 0.03$, $\bar{\rho} \in \{0.03, 0.3, \infty\}$, $g = g_d = h = 0.02$, $k = k_d = 0.9$, $\beta_1 = 0.05$, $\beta_2 = 0.04$, $\beta_3 = 0.02$, $\beta_4 = 0.01$, and $\beta_t = 0$, for $t \geq 5$.

Numerical Analysis

Fair Contract Analysis

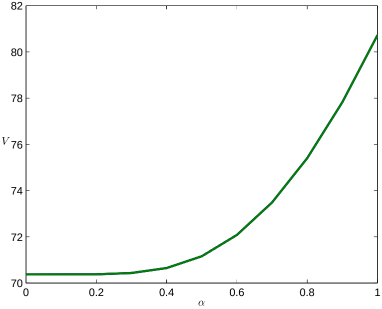

Figure 4: The contract value V_0 depending on the participation rate in the minimum guarantee α for $\underline{\rho} = 0.03$, $\bar{\rho} \in \{0.03, 0.3, \infty\}$, $g = g_d = h = 0.02$, $k = k_d = 0.9$, $\beta_t = 1$, for $t \geq 0$.

Numerical Analysis

Fair Contract Analysis

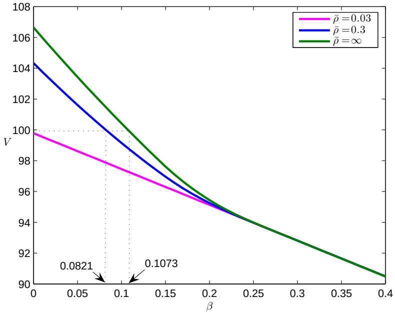

Figure 5: The contract value V_0 depending on the penalty parameter β for $\underline{\rho} = 0.03$, $\bar{\rho} \in \{0.03, 0.3, \infty\}$, $\alpha = 0.85$, $g = g_d = h = 0.02$, $k = k_d = 0.9$.

Numerical Analysis

Fair Contract Analysis

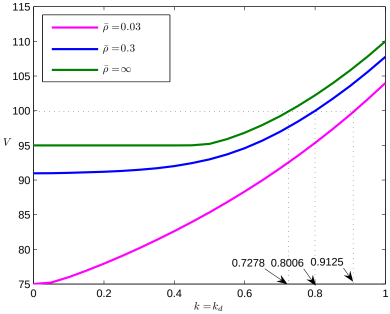

Figure 6: The contract value V_0 depending on the participation rates $k = k_d$ for $\rho = 0.03$, $\bar{\rho} \in \{0.03, 0.3, \infty\}$, $\alpha = 0.85$, $g = g_d = h = 0.02$, $k = k_d = 0.9$, $\beta_1 = 0.05$, $\beta_2 = 0.04$, $\beta_3 = 0.02$, $\beta_4 = 0.01$, and $\beta_t = 0$, for $t \geq 5$.

Numerical Analysis

Fair Contract Analysis

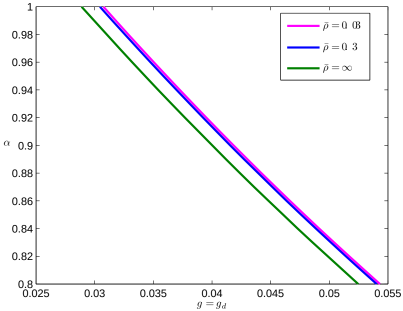

Figure 7: Parameter combinations of the participation rate in the minimum guarantee α and the minimum guaranteed rates at survival and at death $g = g_d$ ensuring a fair contract, for $\underline{\rho} = 0.03$, $\bar{\rho} \in \{0.03, 0.3, \infty\}$, $g = g_d = h = 0.02$, $k = k_d = 0.7$, $\beta_1 = 0.05$, $\beta_2 = 0.04$, $\beta_3 = 0.02$, $\beta_4 = 0.01$, and $\beta_t = 0$, for $t \geq 5$.

Numerical Analysis

Fair Contract Analysis

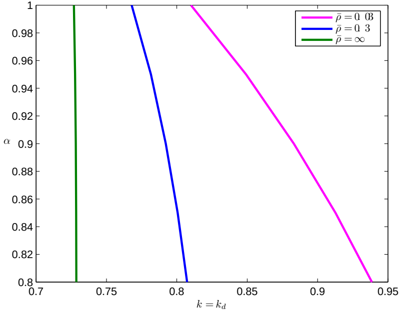

Figure 8: Parameter combinations of the participation rate in the minimum guarantee α and participation rates in the asset performance at survival and at death $k = k_d$ ensuring a fair contract, for $\underline{\rho} = 0.03$, $\bar{\rho} \in \{0.03, 0.3, \infty\}$, $g = g_d = h = 0.02$, $k = k_d = 0.9$, $\beta_1 = 0.05$, $\beta_2 = 0.04$, $\beta_3 = 0.02$, $\beta_4 = 0.01$, and $\beta_t = 0$, for $t \geq 5$.

Numerical Analysis

Fair Contract Analysis

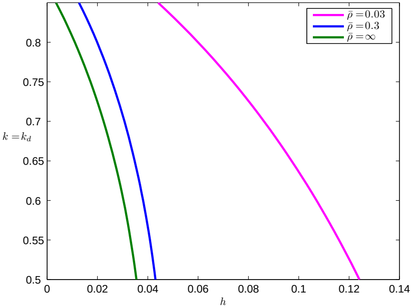

Figure 9: Parameter combinations of the minimum guaranteed rate h for the surrender benefit and the participation rates in the asset performance at survival and at death $k = k_d$ ensuring a fair contract, for $\rho = 0.03$, $\bar{\rho} \in \{0.03, 0.3, \infty\}$, $\alpha = 0.85$, $g = g_d = 0.02$, $\beta_1 = 0.05$, $\beta_2 = 0.04$, $\beta_3 = 0.02$, $\beta_4 = 0.01$, and $\beta_t = 0$, for $t \geq 5$.

Conclusion

- We have studied the valuation of unit-linked life insurance contracts with surrender guarantees at the portfolio level by assuming the surrender intensity to be bounded from below and from above.
- For different degrees of rationality the average contract value can vary significantly. Based on the realistic estimation of their rationality, the contract can be designed more reasonably and an average overvaluation can be avoided.
- We provide the separating boundary between purely exogenous surrender and surrender due to financial reasons. This may help insurance companies to better understand the surrender activity of their policyholders affecting also the companies' hedge programs.
- We provide fair contract analysis with respect to the degree of rationality of the policyholders.

Conclusion

Possible Extensions:

- The bounds $\underline{\rho}$ and $\bar{\rho}$ need not be constant but can be driven by market variables and non-financial factors.
- In a multi-factor model we can apply least-squared Monte Carlo simulation following Longstaff and Schwartz to count in the limited rationality of policyholders. (in progress)
- We can incorporate a secondary market where the policyholder are given the additional option to sell their contract to a third party, and study the impact of a secondary market on contract value and fair contract design. (in progress)

The End

Thanks for Your Attention!